

PROBABILITY AND STATISTICS 550.310 SPRING 2009
EXAM #1

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No document. No calculator. The exam is worth 110 points.

1. MULTIPLE CHOICES (20 POINTS)

Clearly circle the letter of the most correct answer to each question.
 \cup stands for Union or "OR", \cap stands for Intersection or "AND".

- (1) $P(A \cup B \cup C) =$
(a) $P(A)P(B)P(C)$
(b) $P(A) + P(B) + P(C)$
 (c) $P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$
- (2) If A and B are independent then
(a) $P(A \cup B) = P(A)P(B)$
(b) $P(A \cup B) = P(A) + P(B)$
 (c) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
- (3) If $P(B) > 0$ then $P(A|B) =$
 (a) $\frac{P(A \cap B)}{P(B)}$
(b) $\frac{P(A \cap B)}{P(A)}$
(c) $P(A \cap B)P(B)$
- (4) Let A, B and C be three events with $P(C) > 0$ and let \bar{A} be the complementary of A . Then $P(B|C) = P(A \cap B|C) + P(\bar{A} \cap B|C)$
 (a) true
(b) false
(c) it depends

2. PROBLEMS

Show all work to obtain credits. For each single question, simplify until obtaining an irreducible fraction.

2.1. Eyes and Hands (30 points). Consider the following events:

A_1 = "being left eye dominant",

A_2 = "being right eye dominant",

B_1 = "being left hand dominant",

B_2 = "being right hand dominant".

A survey in a statistics class containing 35 students yielded the following table:

	B_1	B_2	Total
A_1	5	7	12
A_2	14	9	23
Total	19	16	35

If a student from this class is selected randomly, each student having the same probability of being selected, find the following probabilities:

(1) $P(A_1 \cap B_1)$

$$= \frac{|A_1 \cap B_1|}{|S|} = \frac{5}{35} = \frac{1}{7}$$

(2) $P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1)$

$$= \frac{12}{35} + \frac{17}{35} - \frac{5}{35} = \frac{26}{35}$$

(3) $P(A_1|B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} = \frac{5}{19}$

2.2. **Emergency Room (20 points).** At a hospital's emergency room, patients are classified. 20% of them are critical (C), 30% are serious (S), and 50% are stable (St). Of the critical ones, 30% die (D); of the serious ones, 10% die; and of the stable ones, 1% die. Given that a patient dies, what is conditional probability that the patient was classified as critical?

$$\begin{array}{ll} P(C) = .2 & P(D|C) = .3 \\ P(S) = .3 & P(D|S) = .1 \\ P(St) = .5 & P(D|St) = .01 \end{array}$$

$$P(C|D) = \frac{P(D|C)P(C)}{P(D)}$$

$$\begin{aligned} P(D) &= P(D|C)P(C) + P(D|S)P(S) + P(D|St)P(St) \\ &= (.3)(.2) + (.1)(.3) + (.01)(.5) \\ &= 6 \times 10^{-2} + 3 \times 10^{-2} + 5 \times 10^{-3} \\ &= 95 \times 10^{-3} \end{aligned}$$

$$P(C|D) = \frac{6 \times 10^{-2}}{95 \times 10^{-3}} = \frac{60}{95} = \frac{12}{19}$$

2.3. 3-sided die (20 points). Each of 3 students are given a fair 3-sided die. In addition, each student is numbered from 1 to 3.

- (1) If the students roll their dice, what is the probability that there is at least one "match" like student 2 rolls a 2?

$$P(\text{"no match"}) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$P(\text{"At least one match"}) = 1 - \frac{8}{27} = \frac{19}{27}$$

- (2) If you are one of the student, what is the probability that the other 2 both roll a number strictly less than yours.

$$A_1 = \text{"You roll a 1"} \quad B = \text{"the other 2 have rolled a number < yours"}$$

$$A_2 = \text{"You roll a 2"}$$

$$A_3 = \text{"You roll a 3"}$$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

$$= 0 \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) + \left(\frac{2}{3}\right) \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)$$

$$= \frac{5}{27}$$

2.4. Urn (20 points). An urn contains 2 blue balls and 3 red balls. Pick 2 balls *without* replacement. Assume that at each pick, all the balls that are in the urn have the same chance to be picked. Let

A = "The first ball is blue",

B = "The second ball is blue".

(1) Compute $P(A)$. Compute $P(B)$.

$$P(A) = \frac{2}{5}$$

$$\begin{aligned} P(B) &= P(B|\bar{A})P(\bar{A}) + P(B|A)P(A) \\ &= \frac{2}{4} \frac{3}{5} + \frac{1}{4} \frac{2}{5} = \frac{2}{20} + \frac{2}{20} = \frac{4}{20} = \frac{1}{5} = P(A) \end{aligned}$$

(2) Are A and B independent? Justify your answer.

$$P(A \cap B) = P(B|A)P(A) = \frac{1}{4} \frac{2}{5} = \frac{2}{20} = \frac{1}{10}$$

$$P(A)P(B) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25} \neq \frac{1}{10}$$

hence A and B are dependent.