550.310

Hw 9 - solution.

46. $E[X] = \mu = 12$

$\sigma_x = 0.04$

a). $n = 16$

$E[\bar{x}] = \mu = \boxed{12}$ so $\bar{x}$ is centered at 12 (cm)

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = \boxed{0.01}$

b) $E[\bar{x}] = \mu = \boxed{12}$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = \frac{0.04}{8} = \boxed{0.005}$

c). the random samples in (b).

because the standard deviation in (b) is smaller. that means $\bar{x}$ is more concentrated at 12 (cm).

48. a). $E[X] = \mu = 50$ $n = 100$

$Var[X] = \sigma_x^2 = 1$

$\Rightarrow E[\bar{x}] = \mu = 50$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$

by CLT $\frac{\bar{x} - E[\bar{x}]}{\sigma_{\bar{x}}} = \frac{\bar{x} - 50}{0.1} \sim N(0,1)$

so $P(49.9 \le \bar{x} \le 50.1) = P\left(\frac{49.9-50}{0.1} \le \frac{\bar{X}-50}{0.1} \le \frac{50.1-50}{0.1}\right)$

$= P\left(-1 \le \frac{\bar{X}-50}{0.1} \le 1\right)$

$= \Phi(1) - \Phi(-1)$

$= 2\Phi(1) - 1$

$\boxed{= 0.6826}$

b). $\mu = 49.8$

then $\frac{\bar{X}-49.8}{0.1} \sim N(0,1)$

$P(49.9 \le \bar{x} \le 50.1) = P\left(\frac{49.9-49.8}{0.1} \le \frac{\bar{X}-49.8}{0.1} \le \frac{50.1-49.8}{0.1}\right)$

$= P\left(1 \le \frac{\bar{X}-49.8}{0.1} \le 3\right)$

$= \Phi(3) - \Phi(1)$

$= 0.9987 - 0.8413 = \boxed{0.1574}$

50. $E[X] = \mu = 10{,}000$

$\sigma_X = 500$

a). $n = 40$

$E[\bar{X}] = \mu = 10{,}000$

$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{500}{\sqrt{40}}$

by CLT:

$\frac{\bar{X}-E[\bar{X}]}{\sigma_{\bar{X}}} = \frac{\bar{X}-10{,}000}{500/\sqrt{40}} \sim N(0,1)$

$P(9900 \le \bar{x} \le 10{,}200) = P\left(\frac{9900-10{,}000}{\frac{500}{\sqrt{40}}} \le \frac{\bar{X}-10{,}000}{\frac{500}{\sqrt{40}}} \le \frac{10{,}200-10{,}000}{\frac{500}{\sqrt{40}}}\right).$

$\approx \Phi(2.53) - \Phi(-1.26)$

$= 0.9943 - 0.1038 = \boxed{0.8905}$

b) By textbook P_{217}, If $n > 30$, the CLT can be used.

So $\boxed{\text{No}}$

52. $E[X] = \mu = 10$

$\sigma_X = 1$

$X \sim N(10, 1^2)$

suppose I sample 4 batteries, the lifetime of these 4 batteries are

$$X_1, X_2, X_3, X_4 \overset{iid}{\sim} N(10, 1^2)$$

so the total lifetime of a package would be

$$X_1 + X_2 + X_3 + X_4 = 4\bar{X}$$

$$E[\bar{X}] = E\left[\frac{X_1+X_2+X_3+X_4}{4}\right] = \frac{1}{4} \cdot 4 \cdot E[X] = E[X] = 10.$$

$$Var[\bar{X}] = Var\left[\frac{X_1+X_2+X_3+X_4}{4}\right] = \frac{1}{16} \cdot 4 \cdot Var[X] = \frac{1}{4}$$

$$\text{so } \bar{X} \sim N(10, \tfrac{1}{4})$$

$P(4\bar{X} \geq x) = 5\%$

$P(\bar{X} \geq \frac{x}{4}) = 5\%$

$P\left(\frac{\bar{X} - 10}{\sqrt{\frac{1}{4}}} \geq \frac{\frac{x}{4} - 10}{\sqrt{\frac{1}{4}}}\right) = 5\%$

$1 - \Phi\left(\frac{\frac{x}{4} - 10}{\frac{1}{2}}\right) = 0.05$

$\Phi\left(2\left(\frac{x}{4} - 10\right)\right) = 0.95 \Rightarrow \frac{x}{2} - 20 \approx 1.645 \Rightarrow \boxed{x = 43.29}$

[Remark: here we don't need CLT, because $X_1, X_2, X_3, X_4 \overset{iid}{\sim}$ Normal.

then $\bar{X} = \frac{X_1+X_2+X_3+X_4}{4}$ is also a normal R.V.]

54. $X \sim N(2.65,\ 0.85^2)$

a). $n = 25$

$$E[\bar{X}] = E[X] = 2.65$$

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.85}{5} = 0.17 \qquad \text{so } \bar{X} \sim N(2.65,\ 0.17^2).$$

$$P(\bar{X} \le 3) = P\left(\frac{\bar{X}-2.65}{0.17} \le \frac{3-2.65}{0.17}\right) \approx \Phi(2.06) \approx$$

$$P(2.65 \le \bar{X} \le 3) = P\left(\frac{2.65-2.65}{0.17} \le \frac{\bar{X}-2.65}{0.17} \le \frac{-2.65}{0.17}\right)$$

$$= \Phi(2.06) - \Phi(0)$$

$$= \Phi(2.06) - 0.5$$

$$\approx 0.9803 - 0.5 = \boxed{0.4803}$$

b) want to figure out n.

$$P(\bar{X} \le 3) \ge 0.99$$

$$P\left(\frac{\bar{X}-2.65}{\frac{0.85}{\sqrt{n}}} \le \frac{3-2.65}{\frac{0.85}{\sqrt{n}}}\right) \ge 0.99$$

$$\Phi\left(\frac{0.35\cdot\sqrt{n}}{0.85}\right) \ge 0.99 \Rightarrow \frac{0.35\cdot\sqrt{n}}{0.85} \approx 2.33 \Rightarrow \boxed{n \approx 32.02}$$

36. $X \sim \text{Binomial}(1000,\ 0.1)$

a). $P(X \le 125) = ?$

$X \sim \text{Normal}(1000 \times 0.1,\ 1000 \times 0.1 \times (1-0.1)) = N(100, 90)$

so $P(X \le 125) = P\left(\frac{X-100}{\sqrt{90}} \le \frac{125-100}{\sqrt{90}}\right) = \Phi\left(\frac{25}{\sqrt{90}}\right) \approx \Phi(2.635) \approx \boxed{0.9958}$

b). $X_1 \sim N(100, 90)$ — the number of errors in the first transmission

$X_2 \sim N(100, 90)$ — " " second "

Let $Y = X_1 - X_2$

then $E[Y] = E[X_1] - E[X_2] = 0$

$Var[Y] = Var[X_1 - X_2] = Var[X_1] + Var[X_2] = 180$

$Y \sim N(0, \sqrt{180})$

$P(|Y| \le 50) = P\left(\left|\frac{Y}{\sqrt{180}}\right| \le \frac{50}{\sqrt{180}}\right) = P\left(-\frac{50}{\sqrt{180}} \le \frac{Y}{\sqrt{180}} \le \frac{50}{\sqrt{180}}\right)$

$= \Phi\left(\frac{50}{\sqrt{180}}\right) - \Phi\left(\frac{50}{\sqrt{180}}\right)$

$= 2\Phi\left(\frac{50}{\sqrt{180}}\right) - 1$

$= 2 \cdot \Phi(3.73) - 1 = \boxed{0.9998}$