550.310

HW 3 solution

2.1 a) $P(\text{"2 or more H"} \mid \text{"At least 1 H"}) = \dfrac{P(\text{"2 or more H"})}{P(\text{"At least 1 H"})}$

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

$= \dfrac{\frac{4}{8}}{\frac{7}{8}} = \dfrac{4}{7}$

b) $P(\text{"2 or more H"} \mid \text{"At least one tail"})$

$= \dfrac{P(\text{"2 H and 1 T"})}{P(\text{"At least 1 T"})} = \dfrac{\frac{3}{8}}{\frac{7}{8}} = \dfrac{3}{7}$

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

2.2.

11	21	31	41
12	22	32	
13	23		
14			

$P(\text{"At least one is 3"} \mid \text{"sum} < 6\text{"})$

$= \dfrac{4}{10}$

$= \dfrac{2}{5}$

2.3 Denote A = "The Queen carries the gene"

B_i = "the i^{th} prince/cess has hemophilia"

$$P(B_4 \mid \bar{B}_1, \bar{B}_2, \bar{B}_3) = \frac{P(\bar{B}_1, \bar{B}_2, \bar{B}_3, B_4)}{P(\bar{B}_1, \bar{B}_2, \bar{B}_3)}$$

def. of conditional probability

$$P(\bar{B}_1, \bar{B}_2, \bar{B}_3) = P(\bar{B}_1, \bar{B}_2, \bar{B}_3 \mid A)P(A) + P(\bar{B}_1, \bar{B}_2, \bar{B}_3 \mid \bar{A})P(\bar{A})$$

law of total probability $= \left(\frac{1}{2}\right)^3 \frac{1}{2} + 1 \times \frac{1}{2}$

$$P(\bar{B}_1, \bar{B}_2, \bar{B}_3, B_4) = P(\bar{B}_1, \bar{B}_2, \bar{B}_3, B_4 \mid A)P(A) + P(\bar{B}_1, \bar{B}_2, \bar{B}_3, B_4 \mid \bar{A})P(\bar{A})$$

$$= \left(\frac{1}{2}\right)^4 \frac{1}{2} + 0 \cdot \frac{1}{2}$$

$$\text{Answer} = \frac{(\frac{1}{2})^5}{(\frac{1}{2})^4 + \frac{1}{2}} = \frac{(\frac{1}{2})^4(\frac{1}{2})}{(\frac{1}{2})^4[1+(\frac{1}{2})^{-3}]} = \frac{1}{2} \cdot \frac{1}{9} = \frac{1}{18}$$

Remark: The B_i's are not independent, but they are independent given A

2.4 A = "age at the time of death"

$P(A>70) = 0.6$, $P(A \geq 80) = 0.2$

$$P(A \geq 80 \mid A > 70) = \frac{P(A \geq 80)}{P(A>70)} = \frac{0.2}{0.6} = \frac{1}{3}$$

2.5 S_0 = "parents do not smoke"

S_1 = "Exactly 1 parent smokes"

S_2 = "2 parents smoke"

A = "having episodes of pneumonia and/or bronchitis in the first year of life".

$P(A|S_0) = 7.8 \times 10^{-2}$; $P(A|S_1) = 11.4 \times 10^{-2}$; $P(A|S_2) = 17.6 \times 10^{-2}$

$P(S_0) = 94 \times 10^{-2}$; $P(S_1) = 4 \times 10^{-2}$; $P(S_2) = 2 \times 10^{-2}$

a) $P(S_2 \cap A) \overset{\text{multiplication rule}}{=} P(A|S_2)\,P(S_2) = 17.6 \times 10^{-2} \times 2 \times 10^{-2} = 35.2 \times 10^{-4} = 3.52 \times 10^{-3}$

b) $P(A) \overset{\text{law of total probability}}{=} P(A|S_2) \cdot P(S_2) + P(A|S_1)\,P(S_1) + P(A|S_0)\,P(S_0)$

$= 3.52 \times 10^{-3} + 11.4 \times 4 \times 10^{-4} + 7.8 \times 94 \times 10^{-4}$

$= 8.14 \times 10^{-2}$

c) $P(S_2|A) \overset{\text{Baye's rule}}{=} \dfrac{P(A|S_2)\,P(S_2)}{P(A)} = \dfrac{3.52 \times 10^{-3}}{8.14 \times 10^{-2}} = 4.3 \times 10^{-2}$

2.6 $H =$ " high-risk"

$M =$ " medium-risk"

$L =$ " Low-risk"

$C =$ " claim over a 10 year period"

$P(H) = 0.1$; $P(M) = 0.2$; $P(L) = 0.7$

$P(C|H) = 0.02$; $P(C|M) = 0.01$; $P(C|L) = 0.0025$

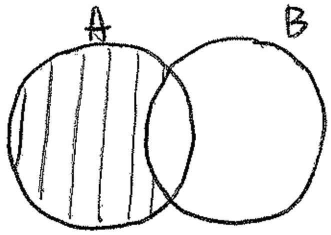
$$P(H|C) = \frac{P(C|H)P(H)}{P(C)}$$

$$P(C) = P(C|H)P(H) + P(C|M)P(M) + P(C|L)P(L)$$

$$= 2\times10^{-3} + 2\times10^{-3} + 17.5\times10^{-6} = 5.75\times10^{-3}$$

$$P(H|C) = \frac{2\times10^{-3}}{5.75\times10^{-3}} = 0.35$$

2.7



$A \cap \bar{B}$

$A = (A\cap\bar{B}) \cup (A\cap B)$ $A\cap\bar{B}$ and $A\cap B$ are mutually exclusive

$P(A) = P(A\cap\bar{B}) + P(A\cap B)$

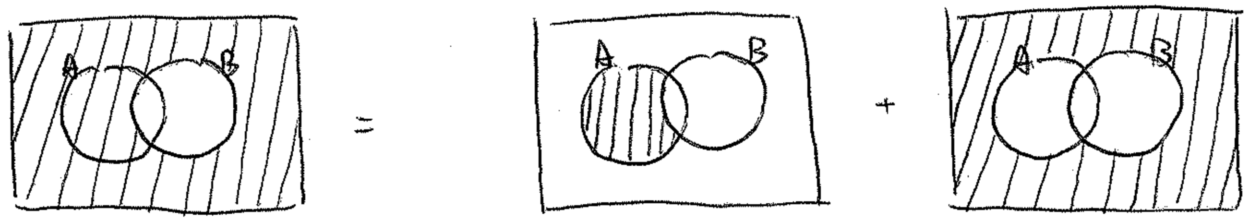
$$\Rightarrow P(A\cap\bar{B}) = P(A) - P(A\cap B)$$

$$= P(A) - P(A)P(B) \quad \text{(by independence of A and B)}$$

$$= P(A)(1 - P(B))$$

$$= P(A)P(\bar{B})$$

So $P(A\cap\bar{B}) = P(A)P(\bar{B})$ A and $\bar{B}$ are independent.



$\bar{B} = (A \cap \bar{B}) \cup (\bar{A} \cap \bar{B})$

$P(\bar{B}) = P(A \cap \bar{B}) + P(\bar{A} \cap \bar{B})$

$\Rightarrow P(\bar{A} \cap \bar{B}) = P(\bar{B}) - P(A \cap \bar{B})$

$= P(\bar{B}) - P(A)P(\bar{B})$ (by independence of A and $\bar{B}$)

$= P(\bar{B})(1 - P(A))$

$= P(\bar{B})P(\bar{A})$

So $P(\bar{A} \cap \bar{B}) = P(\bar{B})P(\bar{A})$ $\quad$ $\bar{A}$ and $\bar{B}$ are independent.

2.8. $p = P($"A single stitch is defective"$)$

$D =$ "the closing is defective"

a) $P(\bar{D}) = 1 - 0.02 = (1-p)^{12}$ $\quad$ solving for p yields 0.001682

b) setting $1 - 0.01 = (1-p)^{12}$ $\quad$ solving for p yields 0.0008372

(because we don't know which stitch is the closing, we'd better consider the opposite of D, which is "none of the 12 stitches are defective")

2.9 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B) = 0.0302$

$P(A)P(B) = 0.0002$

So we need to solve $\begin{cases} ab = 0.0002 \\ a+b = 0.0302 \end{cases} \Rightarrow \begin{cases} P(A) = 0.02039 \\ P(B) = 0.009808 \end{cases}$

2.10 $P(\text{"failure of a criticality 1 item"}) = 10^{-6}$

a) $P(\text{"work"}) = (1-10^{-6})^{748} = 0.928$

b) $1 - 0.928 = 0.072$