150.310

HW2 solution

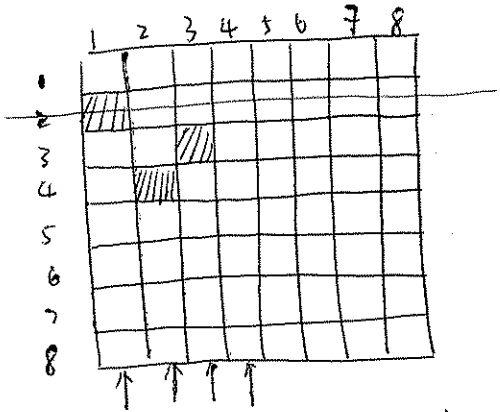
1.1 a) $\frac{\binom{10}{6}}{\binom{45}{6}}$

b) $\frac{\binom{20}{6}+\binom{15}{6}+\binom{10}{6}}{\binom{45}{6}}$

1.2 $\frac{8!}{\binom{64}{8}}$

the denominator is the number of ways to pick 8 out of the 64 squares to have the rooks.

the numerator is the number of ways to put the 8 rooks in different rows and columns.



$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$
choices

1.3. $\frac{7 \times 6 \times 5 \times 4}{10^4}$ (or $\frac{P_{4,7}}{10^4} = \frac{\frac{7!}{(7-4)!}}{10^4}$)

↳ permutation

1.4. $\frac{49 \cdot 48! \cdot 4!}{52!}$ $\left(= \frac{49! \cdot 4!}{52!}\right)$

for the numerator, pick where the aces start, then how the other 48 cards are arranged, and then how the 4 aces are arranged.

1.5 $\frac{\binom{6}{2}\binom{42}{4}}{\binom{48}{6}}$

1.6 $\frac{2! \cdot 2! \cdot 3!}{11!}$

1.7 a) $\frac{\binom{55}{25}+\binom{55}{30}}{\binom{60}{30}}$ since, if you are picking 30 out of the 60 students to make up a particular class, the 5 friends would need to be all in the class or all not in the class.

b) $\frac{\binom{5}{4}\left(\binom{55}{26}+\binom{55}{29}\right)}{\binom{60}{30}}$ since you first have to pick which 4 are in the same class.

1.8 a) for A, it is $\frac{500}{1100}$

for B, it is $\frac{300}{700}$

$\frac{500}{1100} > \frac{300}{700}$

b) for A, it is $\frac{600}{900}$

for B, it is $\frac{900}{1400}$

$\frac{600}{900} > \frac{900}{1400}$

c) for A, it is $\frac{500+600}{2000}$

for B, it is $\frac{300+900}{2100}$

$\frac{500+600}{2000} < \frac{300+900}{2100}$

1.9 a) $\frac{12!}{3! \cdot 3! \cdot 3! \cdot 3!}$

b) $4! / \frac{12!}{3! \cdot 3! \cdot 3! \cdot 3!}$