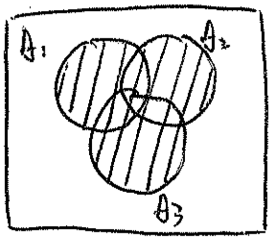
550.310

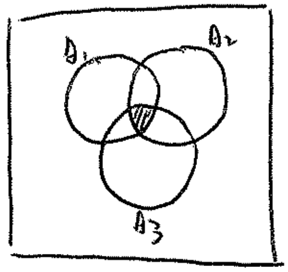
HW 1

1.1 a)



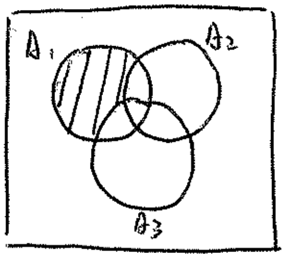
$A_1 \cup A_2 \cup A_3$

b)



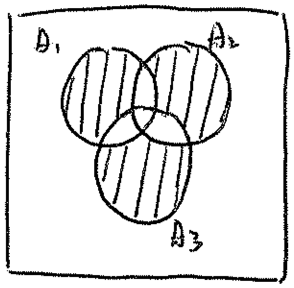
$A_1 \cap A_2 \cap A_3$

c)



$A_1 \cap \bar{A}_2 \cap \bar{A}_3$

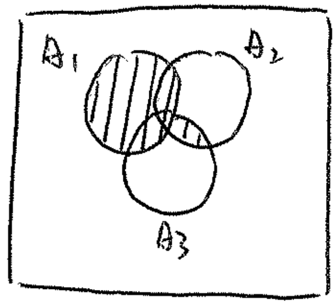
d)



$(A_1 \cap \bar{A}_2 \cap \bar{A}_3) \cup (\bar{A}_1 \cap A_2 \cap \bar{A}_3) \cup (\bar{A}_1 \cap \bar{A}_2 \cap A_3)$

"only site 1 is completed" or "only site 2 is completed" or "only site 3 is completed".

e)



$A_1 \cup (A_2 \cap A_3)$

1.2 In class, we derived from the axioms that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

thus $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$

the last inequality holding since $P(A \cup B) \leq 1$, i.e. $-P(A \cup B) \geq -1$

1.3. Following the hint, we consider events $A_i := E_i \setminus (\cup_{j=1}^{i-1} E_j)$ for $i = 1, 2, 3, \cdots$

Note that $A_1, A_2, A_3, \cdots$ are disjoint; if an outcome is in A_k then it is in E_k, hence not in A_m for any $m < k$, i.e. $A_k \cap A_m = \phi$.

Consequently, by the axioms, $P(\cup_i A_i) = \sum_i P(A_i)$

Also note that $\cup_i A_i = \cup_i E_i$ and note that for all i, $A_i \subseteq E_i$

hence $P(A_i) \leq P(E_i)$.

Thus $P(\cup_i E_i) = P(\cup_i A_i) = \sum_i P(A_i) \leq \sum_i P(E_i)$

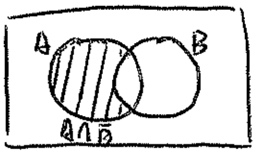
1.4 1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.25 = 0.65$

2. $P(\overline{A \cup B}) = 1 - P(A \cup B) = 0.35$

or $P(\bar{A} \cap \bar{B})$

3. $P(A \setminus B) = P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.5 - 0.25 = 0.25$



1.5 1. $P(A_1 \cup A_2 \cup A_3) = \underbrace{P(A_1 \cup A_2)}_{①} + P(A_3) - \underbrace{P((A_1 \cup A_2) \cap A_3)}_{②}$

$① = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$② = P((A_1 \cap A_3) \cup (A_2 \cap A_3)) = P(A_1 \cap A_3) + P(A_2 \cap A_3) - P((A_1 \cap A_3) \cap (A_2 \cap A_3))$

$= P(A_1 \cap A_3) + P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$

thus, $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3)$

$+ P(A_1 \cap A_2 \cap A_3)$

2. $P(A_1 \cup A_2 \cup A_3 \cup A_4) = \underbrace{P(A_1 \cup A_2 \cup A_3)}_{①} + P(A_4) - \underbrace{P((A_1 \cup A_2 \cup A_3) \cap A_4)}_{②}$

by 1, $① = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3)$
$+ P(A_1 \cap A_2 \cap A_3)$

$② = P((A_1 \cap A_4) \cup (A_2 \cap A_4) \cup (A_3 \cap A_4))$

$\overset{\text{by 1}}{=} P(A_1 \cap A_4) + P(A_2 \cap A_4) + P(A_3 \cap A_4)$
$- P((A_1 \cap A_4) \cap (A_2 \cap A_4)) - P((A_1 \cap A_4) \cap (A_3 \cap A_4)) - P((A_2 \cap A_4) \cap (A_3 \cap A_4))$
$+ P((A_1 \cap A_4) \cap (A_2 \cap A_4) \cap (A_3 \cap A_4))$

$= P(A_1 \cap A_4) + P(A_2 \cap A_4) + P(A_3 \cap A_4)$
$- P(A_1 \cap A_2 \cap A_4) - P(A_1 \cap A_3 \cap A_4) - P(A_2 \cap A_3 \cap A_4)$
$+ P(A_1 \cap A_2 \cap A_3 \cap A_4)$

thus, $P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(A_1) + P(A_2) + P(A_3) + P(A_4)$
$- P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) - P(A_1 \cap A_4) - P(A_2 \cap A_4)$
$- P(A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_4)$
$+ P(A_1 \cap A_3 \cap A_4) + P(A_2 \cap A_3 \cap A_4) - P(A_1 \cap A_2 \cap A_3 \cap A_4)$