550.310

HW 10 - solution

6.1 5.

① $\bar{x} \cdot N = \left(\frac{300+520+526+200+157}{5}\right) \cdot 5000 = \boxed{1{,}703{,}000}$

② $T - N\bar{D} = 1{,}761{,}300 - 5000 \cdot \left(\frac{0+200+0+0-30}{5}\right)$

$= 1{,}761{,}300 - 5000 \cdot 34$

$= \boxed{1{,}591{,}300}$

③ $T \cdot \frac{\bar{X}}{\bar{Y}} = 1{,}761{,}300 \cdot \left(\frac{340.6}{374.6}\right) = \boxed{1{,}601{,}438.28}$

8. a) p = " the proportion that a component is not defective "

$= 1 - \frac{12}{80} = 0.85$

b) P(system will work) = P(both components work)

$= p^2 = \boxed{0.7225}$

10. a). show $\bar{x}^2$ is biased for μ^2

$\boxed{E[\bar{x}^2]} = V[\bar{x}] + (E[\bar{x}])^2 = \frac{\sigma^2}{n} + \mu^2 \boxed{\neq \mu^2}$

b). Let $E[\bar{x}^2 - ks^2] = \mu^2$

$E[\bar{x}^2] - kE[s^2] = \mu^2$ (since $E[s^2] = \sigma^2$)

$\frac{\sigma^2}{n} + \cancel{\mu^2} - k\sigma^2 = \cancel{\mu^2}$ $\Rightarrow \boxed{k = \frac{1}{n}}$

13 $E[\bar{X}] = E[X] = \int_{-1}^{1} 0.5(1+\theta x)\cdot x \cdot dx$

$= \int_{-1}^{1} (\frac{1}{2}x + \frac{1}{2}\theta x^2)\, dx$

$= (\frac{1}{4}x^2 + \frac{1}{6}\theta x^3) \Big|_{-1}^{1}$

$= \frac{1}{3}\theta$

So $\boxed{E(3\bar{x})} = 3E(\bar{x}) = 3\cdot\frac{1}{3}\theta \boxed{= \theta}$

So $3\bar{x}$ is an unbiased estimator for θ.

14. a. Estimate $= \max(X_i) - \min(X_i) + 1 = 525 - 202 + 1 = \boxed{324}$

b. If the same size n equals the total number of Pandemonium jets. Or the CIA happens to catch jet No. α and jet No. β.

The estimate will never be larger than the true total.

since $\max(X_i) \le \beta$

$\min(X_i) \ge \alpha$

$\max(X_i) - \min(X_i) + 1 \le \beta - \alpha + 1$

I think the estimate is biased. Since it is always smaller than or equal to the true value.

18 a). $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(\mu) = \frac{1}{\sqrt{2\pi}\,\sigma}$$

$$\boxed{V(\tilde{x})} = \frac{1}{4n} \cdot [f(\mu)]^{-2} = \frac{1}{4n} \cdot (\sqrt{2\pi} \cdot \sigma)^2 = \frac{1}{4n} \cdot 2\pi \cdot \sigma^2 \boxed{= \frac{\pi}{2n} \cdot \sigma^2}$$

$$\boxed{V(\bar{x}) = \frac{1}{n}\sigma^2}$$

$$V(\tilde{x}) > V(\bar{x})$$

b). Cauchy distribution.

$$f(x) = \frac{1}{\pi\,[1 + (x-\mu)^2]} \qquad -\infty < x < \infty$$ (Textbook P. 236)

$$f(\mu) = \frac{1}{\pi}$$

$$V(\tilde{x}) = \frac{1}{4n} \cdot [f(\mu)]^{-2} = \frac{\pi^2}{4n}$$

$$\lim_{n \to \infty} V(\tilde{x}) = 0$$